

Consider block A of 5 kg , block B of 10 kg , and block C of 10 kg attached by the rope as shown in the figure. All the blocks are in motion. The coefficient of kinetic friction between blocks A and C and the horizontal surface $\mu_{\mathrm{k}}=0.2$

5\% 1- Draw the free body diagrams and kinetic diagrams of blocks A, B, and C.

Solution: $\quad 5(9.81) \mathrm{N}$
A:

$B:$


C:


Constraint
5\% 2- Write the/equation relating the accelerations of blocks $A, B$, and $C$.
Solution:
time derivative

$$
S A+2 S_{B}+S_{C}=\text { Constant }
$$

$$
\begin{aligned}
& V_{A}+2 V_{B}+V_{C}=0 \\
& \vec{a}_{A}+2 \vec{a}_{B}+\vec{a}_{C}=\overrightarrow{0}
\end{aligned}
$$



Based on the assumption of F.B.D.
(B)

$$
-a_{A}+2 a B-a_{c}=0
$$

5\% 3- Apply Newton $2^{\text {nd }}$ Law - F=ma - on blocks A, B, and C.
Solution:

$$
\begin{aligned}
& B: \quad+\quad=F y=\text { may; } 2 T-961=109 B \\
& C: \cos _{+}^{\tan } F_{x}=\max ; T-\underbrace{0.2(10)(9.81)}_{19.62}=10 a_{c} ; \sum F_{y}=0 ; N_{C}=981 \mathrm{~N} \\
& T-59 A=9.81 \\
& 2 T+10 a_{B}=961 \\
& T \text { m }-109 C=19.62
\end{aligned}
$$

5\% 4-Determine the acceleration of each block and the tension in the rope.
Solution:
(1) (2) (3) (4)

$$
\begin{aligned}
\square-a_{A}+2 a_{B}-a_{C} & =0 \\
T-5 a_{A}\left|{ }_{2}\right| 10 a_{B} & =9.81 \\
T & =981 \\
-10 a_{C} & =19.62
\end{aligned}
$$

$$
\begin{aligned}
& T=285.89 \mathrm{~N} \\
& a_{A}=55.216 \mathrm{~m} / \mathrm{s}^{2} \\
& a_{B}=40.922 \mathrm{~m} / \mathrm{s}^{2} \\
& a_{C}=26.627 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$20 \%$
No. 2- (25\%)
In traveling a distance of 3000 m between points $A$ and $D$, a particle is driven at $30 \mathrm{~m} / \mathrm{s}$ from $A$ to $B$ for $t$ seconds and at $15 \mathrm{~m} / \mathrm{s}$ from C to D for $\mathbf{t}$ seconds. The particle uniformly decelerates for 4 seconds between $B$ and $C$.

5\% 1- Determine the deceleration of the particle in the interval BC.
$5 \%$ 2- Determine the distance between $B$ and $C$.

$$
\begin{aligned}
& \stackrel{+}{\Delta} S_{B C}=V_{B} t_{C C}+\frac{1}{2} a_{C} t_{B C}^{2} \\
& \Delta S_{B C}=30 \frac{a_{C}}{2}-\frac{a_{C}}{2}(4)^{2} \\
& V_{C}=V_{B}+a_{C}(4) \\
& 15=30+a_{C}(4) ; a_{C}=\frac{15-30}{4}
\end{aligned}
$$

Solution:

$$
a_{c}=-3.75 \mathrm{~m} / \mathrm{s}
$$

$$
\Delta S_{B C}=30(4)-\frac{3.75}{2}(4)^{2}=90 \mathrm{~m}
$$

$$
\text { or: } 15^{2}-30^{2}=2(-3.75) \triangle S_{B C} ; \Delta S_{B C}=90 \mathrm{~m} \text { Ans. }
$$

5\% 3- Determine the time $\mathbf{t}$ in seconds.
Solution:

$$
t+4+t=t_{A D}
$$

$$
\begin{aligned}
S_{A B}+S_{B C}+S_{C D} & =3000 \\
30 t+90+15 t & =3000 . \quad 45 t=2910 \\
t=\frac{2910}{45} & =64.67 \mathrm{~s} \text { Ans. }
\end{aligned}
$$

$$
\begin{aligned}
& \stackrel{\text { Solution }}{\rightarrow} V_{C}^{V_{B}^{2}}-V_{0}^{2}=2 a_{C} \Delta S_{B C} \\
& (15)^{2}-(30)^{2}=2\left(a_{c}\right) \Delta S_{A B C} \\
& S=S_{0}+V_{0} t ; \quad S_{A B}=30 t \\
& S_{C D}=15 t \\
& S A B+S_{B C}+S_{C D}=3000^{\mathrm{m}}
\end{aligned}
$$

Solution:

$$
\begin{aligned}
& S_{A B}=30 t=30(64.67)=\underline{1940.1 \mathrm{~m}} \text { Ans. } \\
& S_{C D}=15 t=15(64.67)=970.05 \mathrm{~m} \quad \text { Ans. }
\end{aligned}
$$

No. $3-(20 \%)$
The slotted arm revolves in the horizontal plane about the fixed vertical axis through point O . The 2 kg slider C is drawn towards O at the constant rate of $0.05 \mathrm{~m} / \mathrm{s}$ by pulling the cord S . At the instant for which $\mathrm{r}=0.225 \mathrm{~m}$, the arm has a counterclockwise angular velocity at $6 \mathrm{rad} / \mathrm{s}$ and is slowing down at the rate of $2 \mathrm{rad} / \mathrm{s}^{2}$. The slider moves in the smooth slotted arm.

10\% 1- Draw the free body diagram and the kinetic diagram of the slider C in the position shown.

Solution:
 shown.

$$
\begin{aligned}
& \text { Solution: } \begin{aligned}
a_{r} & =\dot{r}^{2}-r \theta^{2}=-0.225\left(6^{2}\right) \\
a_{r} & =-8.1 \mathrm{~m} / \mathrm{s}^{2} \\
a \theta & =r \ddot{\theta}+2 \dot{r} \dot{\theta} \\
& =0.225(-2)+2(-0.05)(6) \\
& =-1.015 \\
\sqrt{a_{\theta}} & =-1.05 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
& \dot{r}=0.225 \mathrm{~m} \\
& \dot{r}=-0.05 \mathrm{~m} / \mathrm{s}=\text { constant } \\
& \dot{r}=0 \\
& \dot{\theta}=6 \mathrm{rad} / \mathrm{s} 5 \\
& \ddot{\theta}=-2 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$

$5 \%$ 3- For the position shown, determine the magnitude of the force exerted on the slider by the side of the smooth radial slot and determine the magnitude of the tension force in the cord.

Solution:

$$
\begin{aligned}
&+\Rightarrow \sum F_{r}=m a r \\
&-T=2 a_{r} \\
&-T= 2(-8.1) \\
& T= 16.2 \mathrm{~N} \text { Ans. }
\end{aligned}
$$

$$
\begin{aligned}
& +_{K} \sum F_{\theta}=m a_{\theta} \\
& F_{D A}=2 a_{\theta} \\
& F_{O A}=2(-1.05)
\end{aligned}
$$

$$
F_{0} A=-2.1 \mathrm{~N} \text { Ans. }
$$

No. $4-(20 \%)$
The 0.2 kg slider moves freely along the fixed curved rod from $A$ to $B$ in the vertical plane under the action of the constant 5 N tension in the curve. The slider is released from rest at B. p cable

$5 \%$ 1- Draw the free body diagram of the slider for an intermediate position between $A$ and $B$.

## Solution:


$5 \%$ 2- Determine the work of the weight on the slider for the motion from $A$ to $B$.
Solution:

$$
\begin{aligned}
U_{W}=-m g \Delta Y & =-0.2(9.81)\left(0.25^{\mathrm{m}}\right) \\
& =-0.4905 \mathrm{~J} \text { Ans. }
\end{aligned}
$$

$10 \%$ 2- Determine the velocity of the slider when it reaches B.
Solution:

$$
T_{1}+\Sigma U_{1}-2=T_{2}
$$

$$
\begin{aligned}
& T_{1}=0 \text { rest } \\
& T_{2}=\frac{1}{2} m V^{2}=\frac{1}{2}(0.2) V^{2}=0.1 \mathrm{~V}^{2} \\
& \sum U_{1-2}=-0.4905 \mathrm{~J}+\left(5^{N}\right)(\Delta l) \\
&=-0.4905+5(0.5)=2.0095 \mathrm{~J}
\end{aligned}
$$

No. $5-(20 \%)$
The velocity and acceleration of a particle are given, for a certain instant, by $\mathbf{v}=(6 \mathbf{i}-3 \mathbf{j}+2 \mathbf{k}) \mathrm{m} / \mathrm{s}$ and $\mathbf{a}=(3 \mathbf{i}-1 \mathbf{j}+5 \mathbf{k}) \mathrm{m} / \mathrm{s}^{2}$.
$V=7$
$a=5.9161$
$10 \% 2$ - Determine the angle between the velocity and the acceleration.
Solution: $\vec{a} \cdot \vec{V}=a \quad V \cos \theta$

$$
(3 \hat{\imath}-1 \hat{\jmath}+5 \hat{k})(6 \hat{\imath}-3 \hat{\jmath}+2 \hat{k})=(5.9161)(7) 80 \theta
$$


$18+3+10=31=(5.9161)(7) \cos \theta$

$$
\begin{gathered}
\cos \theta=\frac{31}{(5.9161)(7)}=0.7486 \\
\theta=\cos ^{-1}(0.7486) \\
\theta=41.53^{\circ}
\end{gathered}
$$

5\% 2- Determine the magnitude of the tangential acceleration.
Solution:

$$
\begin{aligned}
a_{t}=a \cos \theta & =5.9161 \cos 41.53^{\circ} \\
& =4.429 \mathrm{~m} / \mathrm{s}^{2} \text { Ans. }
\end{aligned}
$$

$\hat{u}_{t}=\frac{6}{7} \hat{i}-\frac{3}{7} \hat{\jmath}+\frac{2}{7} \hat{k}$
$\left|\vec{a}_{t}\right|=\vec{a} \cdot \hat{u}_{t}=(3 \hat{\imath}-\hat{\jmath}+5 k) \cdot\left(\frac{6}{7} \hat{\imath}-\frac{3}{7} \hat{\jmath}+\frac{2}{7} \hat{k}\right)$

$$
=\frac{3 \times 6}{7}+3 / 7+\frac{10}{7}=4.429 \mathrm{~m} / \mathrm{s}^{2} \text { Ans. }
$$

5\% 3- Determine the magnitude of the normal acceleration.
Solution:

| $a_{n}$ | $=3.922 \mathrm{~m} / \mathrm{s}^{2}$ Ans |
| ---: | :--- |
| $\vec{a}_{t}=4.429\left[\frac{6}{7} \hat{i}-\frac{3}{7} \hat{j}+\frac{2}{7} \hat{k}\right)$ | $=3.796 \hat{i}-1.896 \hat{j}+1.265 \hat{k} \mathrm{~m} / \mathrm{s}^{2}$ |
| $\vec{a}_{n}=\vec{a}-\vec{a}_{t}$ | $=3 \hat{c}-\hat{j}+5 \hat{k}$ |
|  | $-3.796 \hat{i}+1.89 \hat{j}-1.265 \hat{k}$ |
| $=$ | $-0.796 \hat{i}+0.898 \hat{j}+3.735 \hat{k}$ |
| $a_{n}=\left\|\vec{a}_{n}\right\|=$ | $3.923 \mathrm{~m} / \mathrm{s}^{2}$ Ans 7 |

